Differential Equations

Question 1.

If $(x+2y^3)\frac{dy}{dx}=y$, then

(a) $\frac{x}{y}+y^2=c$ (b) $\frac{y}{x}+x^2=c$

(c) $\frac{x}{y}-y^2=c$ (d) $\frac{y}{x}-x^2=c$

Answer:
(c) $\frac{x}{y}-y^2=c$

Question 2.

The solution of $\frac{dy}{dx}+\frac{y}{x}=\frac{1}{\sqrt{1+x^2}}$ is

(a) $y=\frac{1+x^2}{x}+\frac{c}{x}$ (b) $y=\frac{\sqrt{1+x^2}}{x}+\frac{c}{x}$

(c) $y=\frac{x}{\sqrt{1+x^2}}+cx$ (d) none of these

Answer:
(b) $y=\frac{\sqrt{1+x^2}}{x}+\frac{c}{x}$

Question 3.

The solution of differential equation $\frac{dy}{dx} - 3y = \sin 2x$ is

(a) $y = e^{-3x}\left[\frac{\cos 2x + 3\sin 2x}{13}\right] + c$

(b) $y = e^{-3x}\left(\frac{\cos 2x - 3\sin 2x}{13}\right) + c$

(c) $ye^{-3x} = -e^{-3x}\frac{(2\cos 2x + 3\sin 2x)}{13} + c$

(d) none of these

Answer:

(c) $ye^{-3x} = -e^{-3x}\frac{(2\cos 2x + 3\sin 2x)}{13} + c$

Question 4.

The solution of the differential equation,

$x^2\frac{dy}{dx}.\cos\frac{1}{x} - y\sin\frac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$, is

(a) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$ (b) $y = \frac{x+1}{x\sin\frac{1}{x}}$

(c) $y = \cos\frac{1}{x} + \sin\frac{1}{x}$ (d) $y = \frac{x+1}{x\cos\frac{1}{x}}$

Answer:

(a) $y = \sin\frac{1}{x} - \cos\frac{1}{x}$

Question 5.

The degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is

(a) 1
(b) 2
(c) 3
(d) not defined

Answer:
(d) not defined

Question 6.

The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$ respectively are

(a) 2 and not defined
(b) 2 and 2
(c) 2 and 3
(d) 3 and 3
Answer:
(a) 2 and not defined

Question 7.
Integrating factor of the differential equation

$$(1-x^2)\frac{dy}{dx} - xy = 1 \text{ is}$$

(a) $-x$ (b) $\frac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\frac{1}{2}\log(1-x^2)$

Answer:
(c) $\sqrt{1-x^2}$

Question 8.
Integrating factor of the differential equation $\frac{dy}{dx}$ + y tanx – sec x = 0 is

(a) cos x
(b) sec x
(c) $e^{\cos x}$
(d) $e^{\sec x}$
Answer:
(b) sec x

Question 9.
If $(x + y)^2 \frac{dy}{dx} = a^2$, y = 0 when x = 0, then y = a if $\frac{x}{a}$ =

(a) 1
(b) tan 1
(c) tan 1 + 1
(d) tan 1 – 1
Answer:
(d) tan 1 – 1

Question 10.

If $\frac{dy}{dx} = \sin(x+y) + \cos(x+y), y(0) = 0$, **then**

$\tan\frac{x+y}{2} =$

(a) $e^x - 1$ (b) $\frac{e^x - 1}{2}$ (c) $2(e^x - 1)$ (d) $1 - e^x$

Answer:

(a) $e^x - 1$

Question 11.

If $\sin x \frac{dy}{dx} + y \cos x = x \sin x$, then $(y - 1) \sin x =$

(a) c – x sinx

(b) c + xcosx

(c) c – x cos x

(d) c + x sin x

Answer:

(c) c – x cos x

Question 12.

The solution of differential equation $(e^y + 1) \cos x\, dx + e^y \sin x\, dy = 0$ is

(a) $(e^y + 1) \sin x = c$

(b) $e^x \sin x = c$

(c) $(e^x + 1) \cos x = c$

(d) none of these

Answer:

(a) $(e^y + 1) \sin x = c$

Question 13.

The solution of the differential equation $\frac{dy}{dx} = \frac{x}{1+x^2}$ is

(a) $y = \frac{1}{2}\log|2 + x^2| + c$ (b) $y = \frac{1}{2}\log(1 + x) + c$

(c) $y = \log\left(\sqrt{1 + x^2}\right) + c$ (d) none of these

Answer:

(c) $y = \log(\sqrt{1 + x^2}) + c$

Question 14.

If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$, when $x = 5$, then the value of x when $y = 3$ is

(a) e^5 (b) $e^6 + 1$ (c) $\frac{e^6 + 9}{2}$ (d) $\log_e 6$

Answer:
(c) $\frac{e^6+9}{2}$

Question 15.

If $\frac{dy}{dx} = y \sin 2x, y(0) = 1$ then solution is

(a) $y = e\sin^2 x$ (b) $y = \sin^2 x$

(c) $y = \cos^2 x$ (d) $y = e^{\cos^2 x}$

Answer:
(a) $y = e \sin^2 x$

Question 16.
The differential equation of all 'Simple Harmonic Motions' of given period $\frac{2\pi}{n}$ is

(a) $\frac{d^2x}{dt^2} + nx = 0$ (b) $\frac{d^2x}{dt^2} + n^2x = 0$

(c) $\frac{d^2x}{dt^2} - n^2x = 0$ (d) $\frac{d^2x}{dt^2} + \frac{1}{n^2}x = 0$

Answer:
(b) $\frac{d^2x}{dt^2} + n^2x = 0$

Question 17.
The differential equation of all parabolas whose axes are parallel to y-axis is

(a) $\frac{dy}{dx} = -\frac{c^2}{x^2}$ (b) $\frac{d^2x}{dy^2} = c$

(c) $\frac{d^3y}{dx^3} + \frac{d^2x}{dy^2} = 0$ (d) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = c$

Answer:
(a) $\frac{dy}{dx} = -\frac{c^2}{x^2}$

Question 18.
The Solution of cos(x + y) dy = dx is

(a) $y = \tan\left(\frac{x+y}{2}\right) + C$ (b) $y = \cos^{-1}\left(\frac{y}{x}\right) + C$

(c) $y = x\sec\left(\frac{y}{x}\right) + C$ (d) none of these

Answer:
(a) $y = \tan\left(\frac{x+y}{2}\right) + C$

Question 19.

If $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 1$, then $y =$

(a) $x + \ln x$ (b) $x^2 + x \ln x$
(c) xe^{x-1} (d) $x + x \ln x$

Answer:
(d) x + x ln x

Question 20.

If $(x^2 + y^2)dy = xy\ dx, y(1) = 1$, and $y(x_0) = e$, then $x_0 =$

(a) $\sqrt{2(e^2 - 1)}$ (b) $\sqrt{2(e^2 + 1)}$

(c) $\sqrt{3}.e$ (d) $\sqrt{\frac{e^2+1}{2}}$

Answer:
(c) √3e

Question 21.

If $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}, y(1) = \frac{\pi}{2}$, **then** $y\left(\frac{1}{2}\right) =$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

Answer:
(d) $\frac{\pi}{12}$

Question 22.

If $\frac{dy}{dx} = \frac{y}{x}\left(\frac{x\cos\frac{y}{x} + y\sin\frac{y}{x}}{y\sin\frac{y}{x} - x\cos\frac{y}{x}}\right)$, **then**

(a) $x\cos\frac{y}{x} = cy$ (b) $x\sec\frac{y}{x} = cy$

(c) $\cos\frac{y}{x} = cxy$ (d) $\sec\frac{y}{x} = cxy$

Answer:
(d) $\sec\frac{y}{x} = cxy$

Question 23.

If $\frac{dy}{dx} = \frac{y}{x - \sqrt{xy}}$, **then**

(a) $\sqrt{\frac{x}{y}} = \ln cy$ (b) $-\sqrt{\frac{x}{y}} = \ln cy$

(c) $-2\sqrt{\frac{x}{y}} = \ln cy$ (d) $2\sqrt{\frac{x}{y}} = \ln cy$

Answer:
(c) $-2\sqrt{\frac{x}{y}} = \ln cy$

Question 24.

If $(1+e^{x/y})dx+\left(1-\frac{x}{y}\right)e^{x/y}dy=0$, then

(a) $x-ye^{x/y}=c$ (b) $y-xe^{x/y}=c$

(c) $x+ye^{x/y}=c$ (d) $y+xe^{x/y}=c$

Answer:
(c) $x+ye^{x/y}=c$

Question 25.

The solution curve of $\frac{dy}{dx}=\frac{y^2-2xy-x^2}{y^2+2xy-x^2}$, $y(-1)=1$ is

(a) a straight line (b) parabola
(c) circle (d) ellipse

Answer:
(c) Circle

Question 26.
The differential equation of all circles which pass through the origin and whose centre lies on y-axis is

(a) $(x^2-y^2)\frac{dy}{dx}-2xy=0$ (b) $(x^2-y^2)\frac{dy}{dx}+2xy=0$

(c) $(x^2-y^2)\frac{dy}{dx}-xy=0$ (d) $(x^2-y^2)\frac{dy}{dx}+xy=0$

Answer:
(a) $\left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$

Question 27.
The differential equation of the family of circles touching the x-axis at origin is given by

(a) $y''=\frac{1}{x^2-y^2}y'$ (b) $y'=\frac{2xy}{x^2-y^2}$

(c) $y''-y'=\frac{xy}{x^2-y^2}$ (d) none of these

Answer:
(b) $y' = \frac{2xy}{x^2 - y^2}$

Question 28.
The differential equation representing the family of ellipses with centre at origin and foci on x-axis is given as
(a) xy' + y = 0
(b) $x^2y^2(y'')^2 + yy' = 0$
(c) $xyy'' + x(y')^2 - yy' = 0$
(d) None of these
Answer:
(b) $x^2y^2(y'')^2 + yy' = 0$

Question 29.
The differential equation of all parabolas whose axes are along x-axis is
(a) $y_2^2 + y_1 = 0$
(b) $y_1^2 + y_2 = 0$
(c) $y_1^2 + y_1 y_2 = 0$
(d) $y_1^2 + yy_2 = 0$
Answer:
(d) $y_1^2 + yy_2 = 0$

Question 30.
The equation of family of curves for which the length of the normal is equal to the radius vector is
(a) $y^2 \mp x^2 = k^2$
(b) $y \pm x = k$
(c) $y^2 = kx$
(d) none of these
Answer:
(a) $y^2 \mp x^2 = k^2$

Question 31.
Given the differential equation $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$; y(1) = π
Mark out the correct statement.
(a) solution is $y^2 - \sin y = -2x^3 + C$
(b) solution is $y^2 + \sin y = 2x^3 + C$
(c) $C = \pi^2 + 2\sqrt{2}$
(d) $C = \pi^2 + 2$

Answer:
(b) solution is $y^2 + \sin y = 2x^3 + C$

Question 32.
The differential equation of all parabolas whose axis of symmetry is along the axis of the x-axis is of order
(a) 3
(b) 1
(c) 2
(d) none of these
Answer:
(c) 2

Question 33.
The degree of the equation satisfying the relation
$\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda(\sqrt{1+y^2} - y\sqrt{1+x^2})$ is
(a) 1
(b) 2
(c) 3
(d) none of these
Answer:
(a) 1

Question 34.
The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{2/3} + 4 - \frac{3dy}{dx} = 0$ is
(a) 2
(b) 1
(c) 3
(d) none of these
Answer:
(a) 2

Question 35.
The differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ is (a is a constant)

(a) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$

(b) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(c) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

(d) none of these

Answer:

(b) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

Question 36.

The differential equation satisfied by $y = \frac{A}{x} + B$ is (A, B are parameters)

(a) $x^2 y_1 = y$

(b) $xy_1 + 2y_2 = 0$

(c) $xy_2 + 2y_1 = 0$

(d) none of these

Answer:

(c) $xy_2 + 2y_1 = 0$

Question 37.

The solution of a differential equation is $y = c_1e^{4x} + c_2e^{3x}$, the differential equation is given by

(a) $\frac{d^2y}{dx^1} - 7\frac{dy}{dx} + 7 = 0$ (b) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx}12y = 0$

(c) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$ (d) none of these

Answer:

(c) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$

Question 38.
The differential equation satisfied by

$$\sqrt{1+x^2}+\sqrt{1+y^2}$$

$$=\lambda\left(x\sqrt{1+y^2}-y\sqrt{1+x^2}\right),\lambda\in R \text{ is}$$

(a) $\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$ (b) $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

(c) $\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$ (d) none of these

Answer:
(b) $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

Question 39.
The solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is
(a) $y = \tan^{-1} x$
(b) $y - x = k(1 + xy)$
(c) $x = \tan^{-1} y$
(d) $\tan(xy) = k$
Answer:
(b) $y - x = k(1 + xy)$

Question 40.
The solution of the differential equation cos x sin y dx + sin x cos y dy = 0 is
(a) $\frac{\sin x}{\sin y}=c$
(b) sin x sin y = c
(c) sin x + sin y = c
(d) cos x cos y = c
Answer:
(b) sin x sin y = c

Question 41.
Which of the following is the general solution of

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$?

(a) $y = (Ax + B)e^x$ (b) $y = (Ax + B)e^{-x}$

(c) $y = Ae^x + Be^{-x}$ (d) $y = A\cos x + B\sin x$

Answer:
(a) $y = (Ax + B)\, e^x$

Question 42.

General solution of $\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$ **is**

(a) $y(1+x^2) = c + \tan^{-1} x$ (b) $\frac{y}{1+x^2} = c + \tan^{-1} x$

(c) $y \log(1+x^2) = c + \tan^{-1} x$

(d) $y(1+x^2) = c + \sin^{-1} x$

Answer:
(a) $y\left(1+x^2\right) = c + \tan^{-1} x$

Question 43.

If $\frac{xdy}{dx} - y = \sqrt{x^2+y^2}$ **, then**

(a) $x + \sqrt{x^2+y^2} = cy^2$ (b) $\sqrt{x^2+y^2} - y = cx^2$

(c) $\sqrt{x^2+y^2} + y = cx^2$ (d) $\sqrt{x^2+y^2} - x = cy^2$

Answer:
(c) $\sqrt{x^2+y^2} + y = cx^2$

Question 44.
The solution of the differential equation $(x^2 + y^2)\, dx - 2xy\, dy = 0$ is

(a) $\frac{y}{x^2+y^2}=c$ (b) $\frac{x^2+y^2}{x}=c$

(c) $\frac{y^2-x^2}{y}=c$ (d) $\frac{x^2-y^2}{x}=c$

Answer:
(d) $\frac{x^2-y^2}{x}=c$

Question 45.
The solution of the differential equation x dy + (x + y) dx = 0 is

(a) $c=\frac{y^2}{2}+xy$ (b) $c=xy+\frac{x^2}{2}$

(c) $c=x+\frac{(xy)^2}{2}$ (d) none of these

Answer:
(b) $c=xy+\frac{x^2}{2}$

Question 46.
The solution of differential equation $\frac{dy}{dx}=\frac{x-y}{x+y}$ is
(a) $x^2 - y^2 + 2xy + c = 0$
(b) $x^2 - y^2 - xy + c = 0$
(c) $x^2 - y^2 + xy + c = 0$
(d) $x^2 - y^2 - 2xy + c = 0$
Answer:
(d) $x^2 - y^2 - 2xy + c = 0$

Question 47.
The particular solution $\ln(\frac{dy}{dx}) = 3x + 4y$, y(0) = 0 is
(a) $e^{3x} + 3e^{-4y} = 4$
(b) $4e^{3x} - 3e^{-4y} = 3$
(c) $3e^{3x} + 4e^{4y} = 7$
(d) $4e^{3x} + 3e^{-4y} = 7$
Answer:
(d) $4e^{3x} + 3e^{-4y} = 7$

Question 48.
The solution of the differential equation

$$\frac{x}{x^2+y^2}dy=\left(\frac{y}{x^2+y^2}-1\right)dx\text{, is}$$

(a) $y=x\cot(C-x)$ (b) $\cos^{-1}\frac{y}{x}=(-x+C)$

(c) $y=x\tan(C-x)$ (d) $\frac{y^2}{x^2}=x\tan(C-x)$

Answer:
(c) y = x tan(C – x)

Question 49.
The solution of the differential equation

$$\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)\text{, when } x=1, y=1\text{, is}$$

(a) $\log\left|\frac{(x-y)^2-2}{2}\right|=2(x+y)$

(b) $\log\left|\frac{(x-y)^2+2}{2}\right|=2(x-y)$

(c) $\log\left|\frac{(x+y)^2+2}{2}\right|=2(x-y)$

(d) none of these

Answer:
(d) None of these

Question 50.
The solution of the differential equation

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0 \text{ , is}$$

(a) $y = x\tan\left(\frac{x^2 + y^2 + C}{2}\right)$

(b) $x = y\tan\left(\frac{x^2 + y^2 + C}{2}\right)$

(c) $y = x\tan\left(\frac{C - x^2 - y^2}{2}\right)$

(d) none of these

Answer:

(c) $y = x\tan\left(\frac{C - x^2 - y^2}{2}\right)$

Question 51.

$$\text{If } \frac{dy}{dx} = \frac{2}{x + y} \text{, then } x + y + 2 =$$

(a) ce^y (b) $ce^{y/2}$ (c) ce^{-y} (d) $ce^{-\frac{y}{2}}$

Answer:

(b) $ce^{y/2}$

Question 52.

The differential equation $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{y}}$ determines a family of circle with

(a) variable radii and fixed centre (0, 1)

(b) variable radii and fixed centre (0, -1)

(c) fixed radius 1 and variable centre on x-axis

(d) fixed radius 1 and variable centre on y-axis

Answer:

(c) fixed radius 1 and variable centre on x-axis

Question 53.

If $y\,dx + y^2\,dy = x\,dy$, $x \in R$, $y > 0$ and $y(1) = 1$, then $y(-3) =$

(a) 3

(b) 2
(c) 1
(d) 5
Answer:
(a) 3

Question 54.
The solution of $y\,dx + (x + x^2y)\,dy = 0$ is

(a) $-\frac{1}{xy} = c$ (b) $-\frac{1}{xy} + \ln y = c$

(c) $\frac{1}{xy} + \ln y = c$ (d) $\ln y = cx$

Answer:
(b) $-\frac{1}{xy} + \ln y = c$

Question 55.
If $\frac{xdy}{dx} + 2y = \ln x$, then $e^2 y(e) - y(1) =$

(a) $\frac{e^2+1}{2}$ (b) $\frac{e^2+1}{3}$ (c) $\frac{e^2+1}{4}$ (d) $e^2 + 1$

Answer:
(c) $\frac{e^2+1}{4}$

Question 56.
If $x(x-1)\frac{dy}{dx} - y = x^2(x-1)^2$, then $4y(2) - y(1) =$

(a) 0 (b) 2 (c) 4 (d) 6

Answer:
(d) 6

Question 57.
If $x \ln x \frac{dy}{dx} + y = 2\ln x, y(e) = 2$, then $y(e^2) =$

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$

Answer:
(d) $\frac{5}{2}$

Question 58.

If $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x, y(0)=1$, then $y\left(\frac{\pi}{4}\right)=$

(a) $\frac{1}{e}$ (b) e (c) $2e$ (d) $\frac{2}{e}$

Answer:
(d) $\frac{2}{e}$